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DISCUSSION ON TEMPERATURE DISTRIBUTION IN CHANNEL FLOW WITH FRICTION

J. MADEJSKI, *Int. J. Heat Mass Transfer* **6**, 49 (1963)

SIAVASH ESHGHY

Carnegie Institute of Technology, Pittsburgh, Pennsylvania 15213

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IN A RECENT paper, J. Madejski states that "the usually neglected pressure drop in the energy equation must be taken into account if the heat of friction is not negligible. . . ." The author bases his reasoning on an energy equation of the form

$$\rho C_p \frac{dT}{dt} = \frac{dp}{dt} + \mu\phi + \nabla . (\kappa \nabla T) \tag{1}$$

and then argues that dp/dt and $\mu\phi$ are usually of the same order of magnitude, therefore, whenever $\mu\phi$ is considered dp/dt should also be maintained. He then proceeds to calculate the temperature distribution for fully-developed flow in channels and tubes including the dp/dt term in the energy equation. His results are in contrast with the classical solution of the same problem.

It is felt that the assumptions in the above analysis are unrealistic and the results thereof, based on the following reasons, questionable.

1. The energy equation used [equation (1)] is only valid for an ideal gas. The more general form of the energy equation may be found in references 2 and 3 as

$$\rho C_p \frac{dT}{dt} = \beta T \frac{dp}{dt} + \mu\phi + \nabla . (\kappa \nabla T) \tag{2}$$

where

$$\beta = -\frac{1}{\rho}\left(\frac{\partial \rho}{\partial T}\right)_p$$

is the coefficient of volume expansivity at constant pressure whose value is $1/T$ for an ideal gas which reduces equation (2) to (1). For a truly incompressible fluid $\beta = 0$, therefore, the term involving pressure gradients drops out altogether. Consequently, the absence of this term in the classical approach to this problem is consistent with the assumption of the fluid being incompressible.

2. As mentioned in reference 1, dp/dt is of the same order of magnitude as $\mu\phi$ in pipe and channel flow. Therefore, whenever the term $\mu\phi$ is significant, the term involving the pressure gradient should be retained but only if βT is of the order of unity or higher. For liquids, however, βT is much less than unity except near critical states. For water at 60°F, for instance, $\beta T = 0{\cdot}044$, and 0·19 for light oil at the same temperature.

3. Fully-developed flow in pipes and channels, referred to in reference 1, will not be accomplished under any arbitrary thermal and flow conditions unless the fluid is truly a constant-property one. For real liquids fully-developed flow will be accomplished only if the thermal boundary conditions are chosen such that temperature profile develops. This is true due to the fact that liquid properties are insensitive to pressure variations. The above set of conditions is no longer sufficient if the fluid is gaseous. The density of gases varies significantly with pressure and fully-developed flow requires a finite pressure gradient. Therefore, no matter how small the inlet Mach number, precisely speaking, fully-developed flow will never be accomplished. Nevertheless, one may speak of fully-developed flow of gases when the pressure gradient is small and hence viscous dissipation $\mu\phi$ is negligible. Flow of gases in channels and pipes under conditions where $\mu\phi$ and dp/dt are significant, is highly compressible and should be treated accordingly. Under such extreme conditions the assumption of fully-developed flow is quite unrealistic.

In conclusion, the analysis in reference 1 is not valid for liquids since the energy equation used is only valid for ideal gases, neither is it applicable to gases due to the unrealistic assumption of fully-developed velocity profile.

REFERENCES

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